Travelling waves in laterally-inhibited grids of integrate-and-fire neurons

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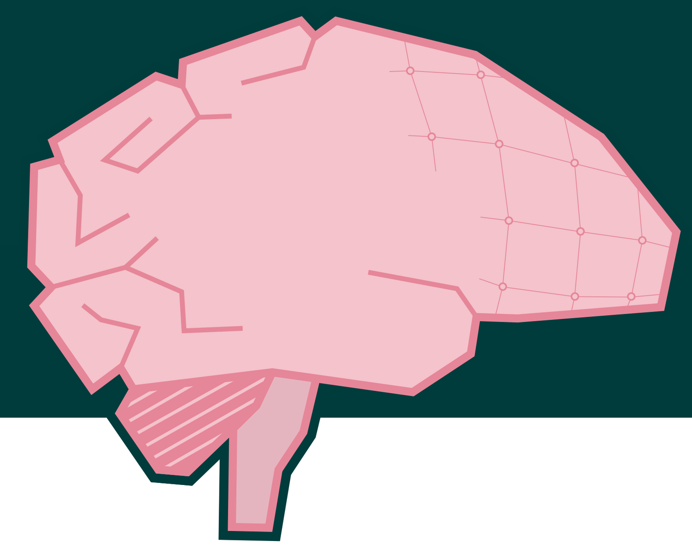


Model construction

We are investigating the propagation of electrical activity through brain tissue by modelling travelling waves moving through a lattice of spiking neurons. We make use of a leaky integrate-and-fire model with subthreshold ion channel resonances[1,2,3]:

Cell voltage:

$$\frac{\partial v}{\partial t} = I - v - u + s - (v_{th} - v_r)\sum_{n=1}^{\infty}\delta(t - t_n(x))$$

When v reaches v_{th}, it resets to v_r and the neuron fires.

Ion channel adaptation:

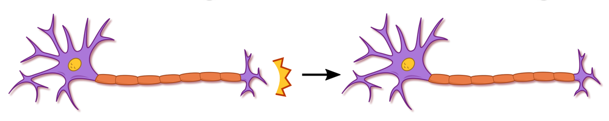
$$\frac{\partial u}{\partial t} = Rv - Du$$

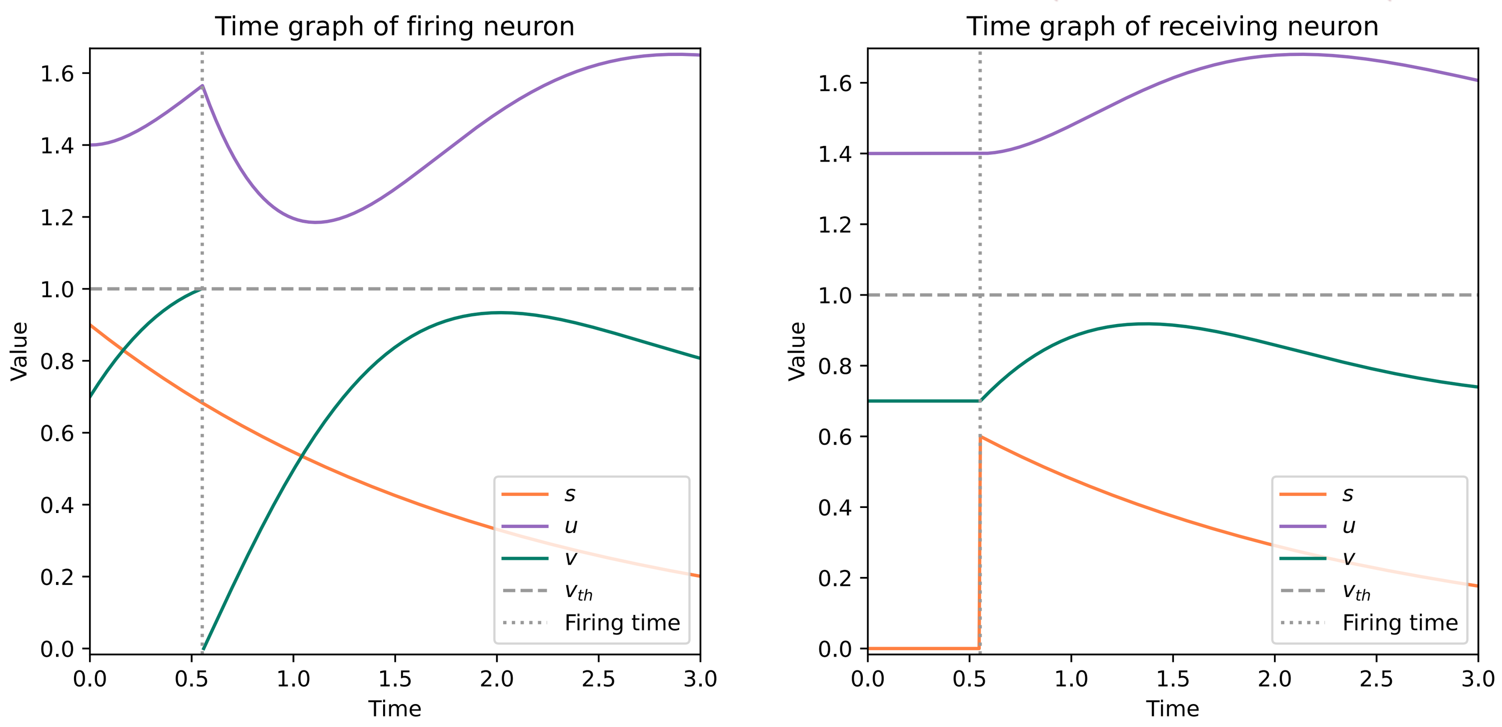
Synaptic input buffer:

$$\frac{\partial s}{\partial t} = -\beta s + \beta\sum_{n=1}^{\infty}\int_{\Omega} w(x-y)\,\delta(t - t_n(y))\,dy$$

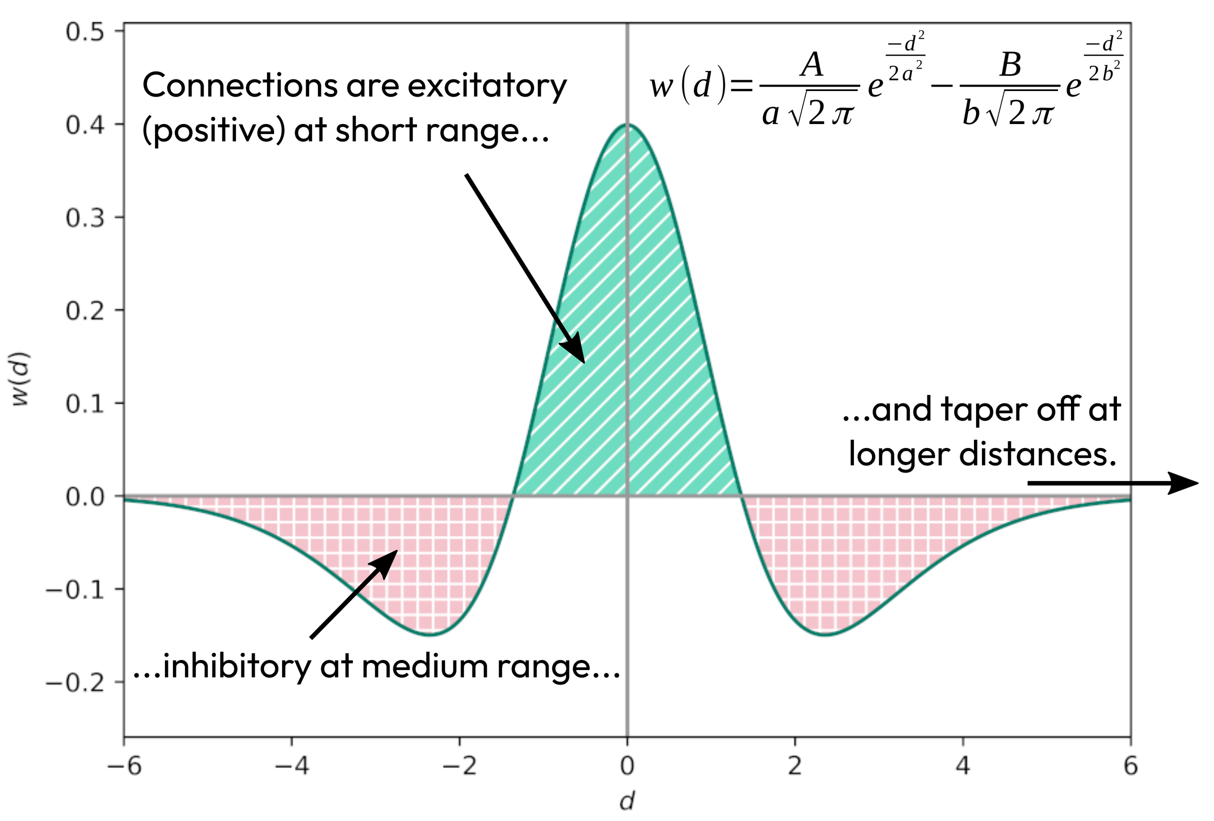
Inputs from firing neurons are integrated across space using a connectivity function w.

These figures give an example of two neurons firing and receiving a signal respectively in a discrete context:

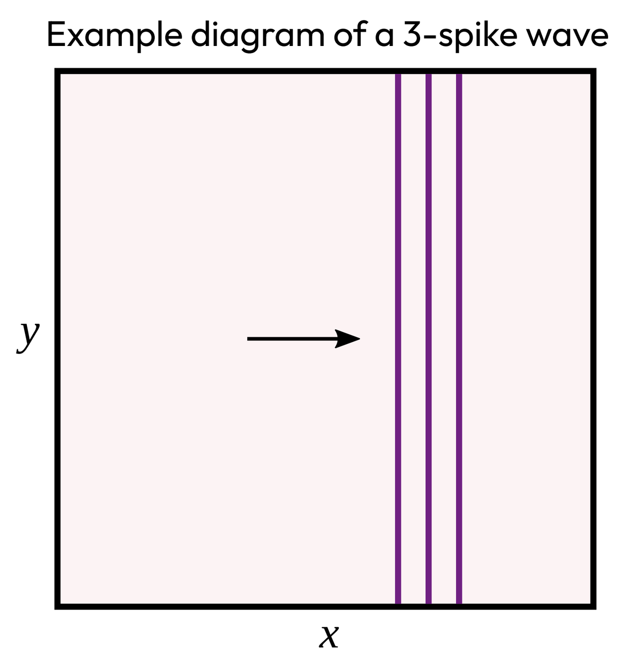




The change in s due to an incoming firing signal is determined by the connection function w based upon the distance between the neurons. We choose w to be a Mexican Hat function constructed from a difference of Gaussians, giving both excitation and inhibition.

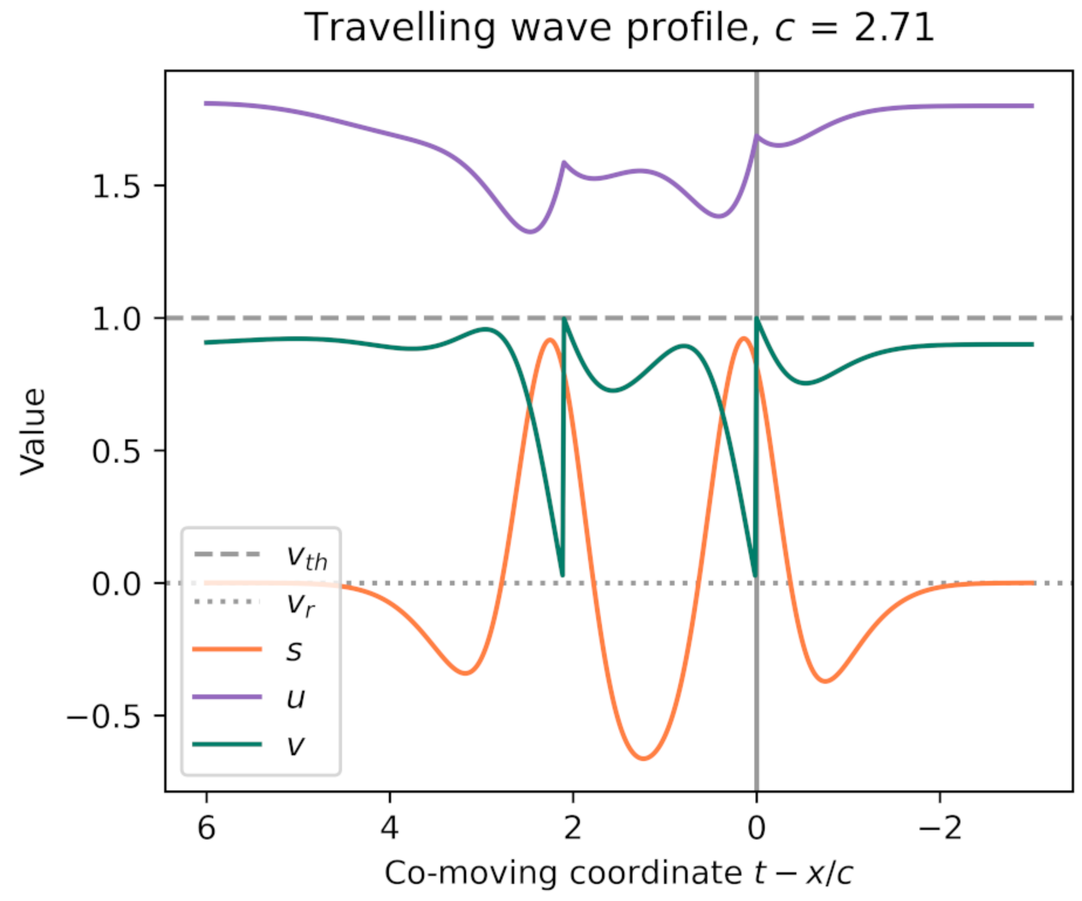


This class of model is known to support travelling waves[1]. In particular, we study planar waves, where concurrent firing events form straight lines, or parallel sets of straight lines, moving at constant speed across the domain.

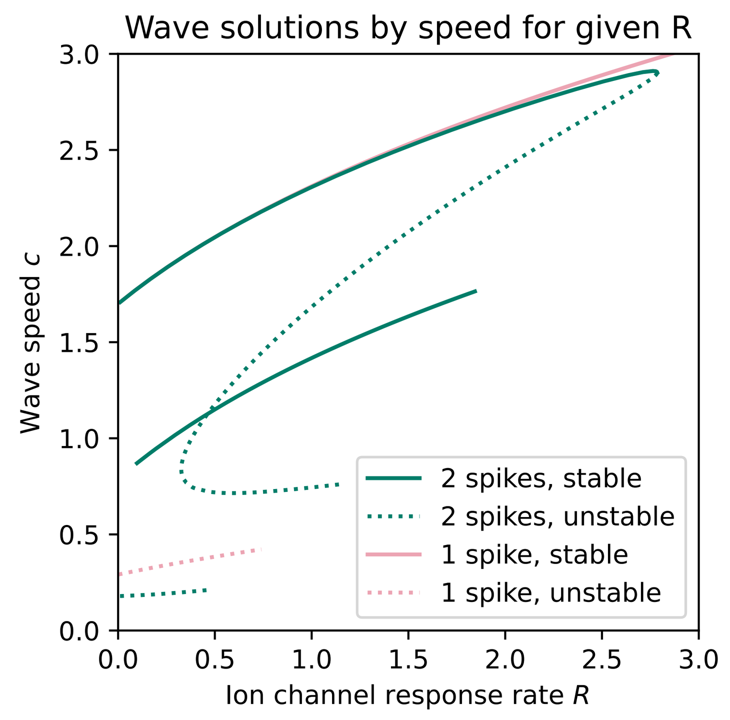


Resonance

While we perform computer simulations of the system with discrete neurons, for analysis we take a continuum limit in which we can use root-finding on the firing threshold crossings to find travelling waves solutions for v, u, s that are constant in co-moving coordinates.

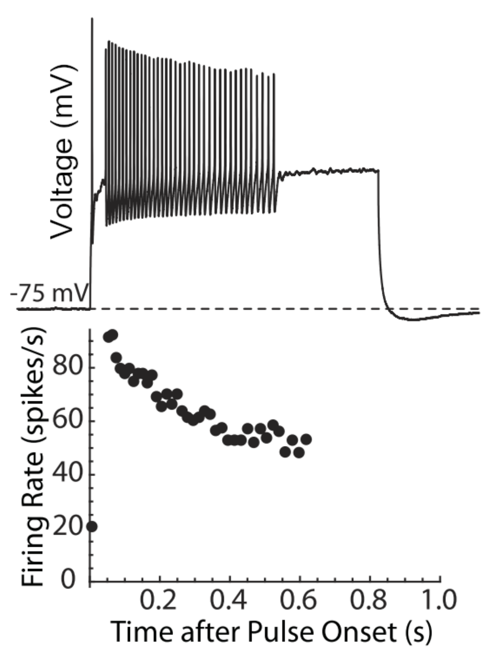


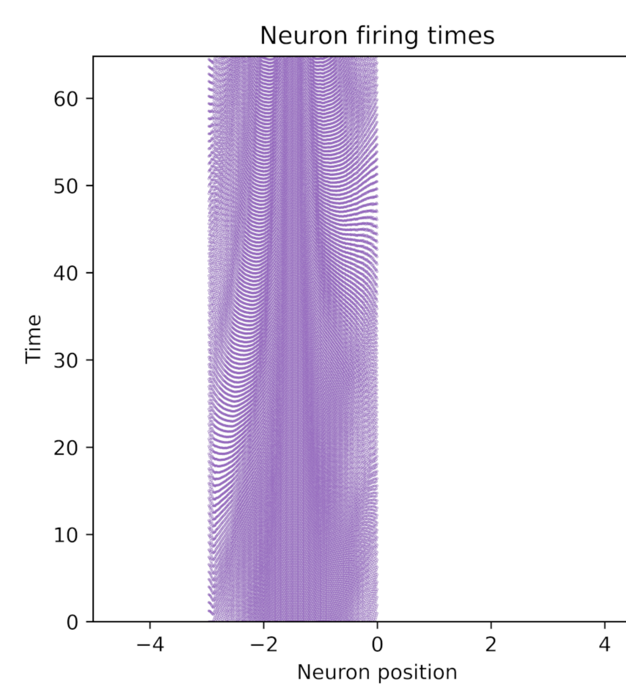
As a wave approaches a given neuron, it applies inhibition then excitation, decreasing then increasing v. As an adaptation variable, u acts to oppose changes in v, but the delay in its response leaves a window during which both u and the synaptic input are acting to increase v, resulting in a more excitable neuron. The strength of response in u thus increases wave speed.



This figure gives wave speeds for 1- and 2-spike planar waves for given values of R, the response rate of u to changes in v. The input current I is adjusted to maintain the rest value of v. Branch termination occurs when a solution gives too many or too few threshold crossings.

If the rest state of v is high within the interval $[v_r, v_{th}]$ then during repeated firing the average value of v is below rest, causing an overall decrease in u that encourages further firing until u is able to recover. Similar bursting behaviour has been observed experimentally, as in the figure (left) below[4]. Repeated firing also anchors the Mexican Hat, producing slow waves that can fail to move at all in the discrete model (right; one-dimensional).





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[3] H. G. Rotstein & F. Nadim, *Frequency preference in two-dimensional neural models: a linear analysis of the interaction between resonant and amplifying currents*, Journal of Computational Neuroscience, 2014

[4] G. Sciamanna & C. J. Wilson, *The ionic mechanism of gamma resonance in rat striatal fast-spiking neurons*, Journal of Neurophysiology, 2011